

<b>REPORT DOCUMENTATION PAGE</b>			<i>Form Approved</i> OMB NO. 0704-0188	
Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comment regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.				
1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE <b>8/19/96</b>		3. REPORT TYPE AND DATES COVERED <b>Final Progress Report 3/93 - 3/96</b>
4. TITLE AND SUBTITLE  <b>Numerical Methods in Stochastic Control</b>			5. FUNDING NUMBERS  <b>DAAH04 - 93 - G - 0070</b>	
6. AUTHOR(S)  <b>Paul Dupuis and Harold J. Kushner</b>				
7. PERFORMING ORGANIZATION NAMES(S) AND ADDRESS(ES) <b>Lefschetz Center for Dynamical Systems Division of Applied Mathematics Brown University Providence, RI 02912</b>			8. PERFORMING ORGANIZATION REPORT NUMBER	
9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)  U.S. Army Research Office P.O. Box 12211 Research Triangle Park, NC 27709-2211			10. SPONSORING / MONITORING AGENCY REPORT NUMBER  <i>ARO 30815.1-MA</i>	
11. SUPPLEMENTARY NOTES  The views, opinions and/or findings contained in this report are those of the author(s) and should not be construed as an official Department of the Army position, policy or decision, unless so designated by other documentation.				
12a. DISTRIBUTION / AVAILABILITY STATEMENT  Approved for public release; distribution unlimited.			12 b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words)  <b>The project concerns the development of effective numerical methods for the solution of stochastic control problems. In order to focus on fundamental issues, the bulk of the model applications are taken from current problems in high speed telecommunications, but the results are applicable in stochastic control in general.</b>				
14. SUBJECT TERMS			15. NUMBER OF PAGES	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OR REPORT <b>UNCLASSIFIED</b>		18. SECURITY CLASSIFICATION OF THIS PAGE <b>UNCLASSIFIED</b>		19. SECURITY CLASSIFICATION OF ABSTRACT <b>UNCLASSIFIED</b>
				20. LIMITATION OF ABSTRACT  <b>UL</b>

19960910 023

DTIC QUALITY INSPECTED 3

## GENERAL INSTRUCTIONS FOR COMPLETING SF 298

The Report Documentation Page (RDP) is used in announcing and cataloging reports. It is important that this information be consistent with the rest of the report, particularly the cover and title page. Instructions for filling in each block of the form follow. It is important to ***stay within the lines*** to meet ***optical scanning requirements***.

### **Block 1. Agency Use Only (Leave blank)**

**Block 2. Report Date.** Full publication date including day, month, and year, if available (e.g. 1 Jan 88). Must cite at least year.

**Block 3. Type of Report and Dates Covered.** State whether report is interim, final, etc. If applicable, enter inclusive report dates (e.g. 10 Jun 87 - 30 Jun 88).

**Block 4. Title and Subtitle.** A title is taken from the part of the report that provides the most meaningful and complete information. When a report is prepared in more than one volume, repeat the primary title, add volume number, and include subtitle for the specific volume. On classified documents enter the title classification in parentheses.

**Block 5. Funding Numbers.** To include contract and grant numbers; may include program element number(s), project number(s), task number(s), and work unit number(s). Use the following labels:

<b>C</b> - Contract	<b>PR</b> - Project
<b>G</b> - Grant	<b>TA</b> - Task
<b>PE</b> - Program Element	<b>WU</b> - Work Unit Accession No.

**Block 6. Author(s).** Name(s) of person(s) responsible for writing the report, performing the research, or credited with the content of the report. If editor or compiler, this should follow the name(s).

**Block 7. Performing Organization Name(s) and Address(es).** Self-explanatory.

**Block 8. Performing Organization Report Number.** Enter the unique alphanumeric report number(s) assigned by the organization performing the report.

**Block 9. Sponsoring/Monitoring Agency Name(s) and Address(es).** Self-explanatory.

**Block 10. Sponsoring/Monitoring Agency Report Number.** (If known)

**Block 11. Supplementary Notes.** Enter information not included elsewhere such as; prepared in cooperation with...; Trans. of...; To be published in.... When a report is revised, include a statement whether the new report supersedes or supplements the older report.

**Block 12a. Distribution/Availability Statement.** Denotes public availability or limitations. Cite any availability to the public. Enter additional limitations or special markings in all capitals (e.g. NORFORN, REL, ITAR).

**DOD** - See DoDD 4230.25, "Distribution Statements on Technical Documents."

**DOE** - See authorities.

**NASA** - See Handbook NHB 2200.2.

**NTIS** - Leave blank.

### **Block 12b. Distribution Code.**

**DOD** - Leave blank

**DOE** - Enter DOE distribution categories from the Standard Distribution for Unclassified Scientific and Technical Reports

**NASA** - Leave blank.

**NTIS** - Leave blank.

**Block 13. Abstract.** Include a brief (*Maximum 200 words*) factual summary of the most significant information contained in the report.

**Block 14. Subject Terms.** Keywords or phrases identifying major subjects in the report.

**Block 15. Number of Pages.** Enter the total number of pages.

**Block 16. Price Code.** Enter appropriate price code (*NTIS only*).

**Block 17. - 19. Security Classifications.** Self-explanatory. Enter U.S. Security Classification in accordance with U.S. Security Regulations (i.e., UNCLASSIFIED). If form contains classified information, stamp classification on the top and bottom of the page.

**Block 20. Limitation of Abstract.** This block must be completed to assign a limitation to the abstract. Enter either UL (unlimited) or SAR (same as report). An entry in this block is necessary if the abstract is to be limited. If blank, the abstract is assumed to be unlimited.

**NUMERICAL METHODS IN STOCHASTIC CONTROL**

**FINAL PROGRESS REPORT**

**HAROLD J. KUSHNER  
PAUL DUPUIS**

**AUGUST 19, 1996**

**U.S. ARMY RESEARCH OFFICE**

**LEFSCHETZ CENTER FOR DYNAMICAL SYSTEMS  
DIVISION OF APPLIED MATHEMATICS  
BROWN UNIVERSITY  
PROVIDENCE, R.I. 02912**

**ARO CONTRACT/GRANT NUMBER: DAAH04-93-G-0070**

**APPROVED FOR PUBLIC RELEASE;**

**DISTRIBUTION UNLIMITED**

**THE VIEWS, OPINIONS, AND/OR FINDINGS CONTAINED IN THIS  
REPORT ARE THOSE OF THE AUTHOR(S) AND SHOULD NOT BE  
CONSTRUED AS AN OFFICIAL DEPARTMENT OF THE ARMY POSITION,  
POLICY, OR DECISION, UNLESS SO DESIGNATED BY OTHER  
DOCUMENTATION.**

# Numerical Methods in Stochastic Control

Harold J. Kushner and Paul Dupuis  
Brown University  
Applied Mathematics  
Providence RI 02912

## The problem studied

The project concerns the development of effective numerical methods for the solution of stochastic control problems. In order to focus on fundamental issues, the bulk of the model applications are taken from current problems in high speed telecommunications, but the results are applicable in stochastic control in general.

## Results

Two categories of problems are of interest. The first concerns large deviations methods for controlled queueing networks of the type that arise in communication. Desired error rates (i.e., noise effects) are generally exceedingly small, cannot be estimated directly, and require the methods of large deviation theory. Numerical methods are essential to get good estimates on performance, and to enable good designs. Our numerical methods are the only ones available at this time. The second category of problems is the optimization of applications which can be approximated by controlled diffusions, or diffusion or jump diffusion processes, as is the case for a large number of the telecommunications problems of interest. This approximation is possible for a large number of important problems in the subject areas. Our Markov chain approximation method is the current method of choice. The original problem is approximated by a computable problem which is still physically close to it. Efficient algorithms codes have been developed and tested on practical problems with excellent results. More detail follows.

The first set of problems studied involves the analysis of and development of numerical methods for communication and computer networks, where the noise effects are small. It is frequently necessary that the "error" rates of the system be quite small. An "error" might be a blocked transmission or loss of a data packet (say, with probability of order  $10^{-8}$  or smaller). Direct calculation of such probabilities is memory intensive and slow, if not impossible. In addition, such calculations are not very informative, since they provide only a mass of numbers with little information on how different parts of the network interact. The goal of the analysis is often to understand how errors can be avoided, and for design or regulation purposes one needs more information. Direct simulation in such cases is also usually out of the question, and in any case it would not give the qualitative information needed for design. Since experience dictates that the probabilities will scale in an exponential fashion with respect to a parameter that measures the "capacity" of the system, e.g, the buffer sizes, an

asymptotic (large deviations) approach is natural. These techniques are naturally suited to the difficult types of problems where rare events are important, and conventional intuition not very useful. The large deviations approximations are defined in terms of the solution to a variational problem, and it is the minimizing parameter in this variational problem that identifies the most likely way in which the undesirable event can occur. However, the discontinuities and other nonstandard features of communication networks make the large deviation analysis of such systems very difficult, and relatively few results are available for networks. These quantities are hard to compute and numerical methods are essential. Their development is based on an alternative and powerful approach to the general theory of large deviations, which also facilitates the analysis of the networks themselves.

A key ingredient in our approach to large deviations are representations for the normalized quantities of interest. Since large deviation expectations and probabilities scale in an exponential fashion, it is natural to phrase the convergence statement in terms of the normalized logarithm of the quantity of interest. The representations facilitate this analysis significantly, and allow one to substitute the well developed methods of weak convergence theory for the relatively awkward exponential estimates used in standard approaches. The representation is as the solution to a variational problem, and in all cases it takes the form of a small noise stochastic control problem. Besides their use in the large deviation analysis, the representations are also important because they provide the starting point for approximations. Furthermore, the methods used for the large deviations convergence can frequently be applied to prove convergence of these approximations.

The paper [A] and the book [B] develop representations for a wide variety of stochastic systems. [B] focuses largely on discrete time processes in a general framework, while [A] is devoted to continuous time Markov models for queueing networks. Besides setting up the representations, [A] proves their convergence under weak conditions and provides a means for identifying the limit problem variational problem.

The paper [C] develops the theory for networks in a different direction. Instead of laying the foundations for a very general theory, this paper concentrates on a particular non-Markovian network model that is currently of interest in the context of high-speed data networks. It is the first and only paper to date that allows for a complete large deviations analysis of bursty data sources in a network environment. In particular, in [C] the variational problem that characterizes the large deviation approximation is solved explicitly, thus providing a concrete characterization of how the interactions between different parts of the network lead to significant data loss. The methods can be generalized to other network and data models.

Lastly there is the paper [D], which is concerned with rates of convergence for approximations in stochastic and deterministic control. The paper presents a simple method for obtaining rate of convergence which is applicable to a wide

range of approximation problems. Heavy use is made of representations of the approximations as functionals of a controlled Markov chain, which of course is a by-product of the Markov chain approximation method. The method is illustrated through the presentation of a number of examples, including finite difference schemes for stochastic and deterministic optimal control problems with several different cost structures. A general principle can be abstracted, and indeed the method may be applied to a variety of approximation problems, such as the numerical approximation of nonlinear PDE not a priori related to control theory.

Concerning the second class of problems, codes have been written, and applied very successfully to problems in telecommunications, yielding important information on the performance and design which is not available by other means.

We have developed domain decomposition methods for the numerical solution of Markov chain control problems on large state spaces of the type which arise as approximations to optimal stochastic control problems with diffusion, jump-diffusion or reflected diffusion type models. These are also applicable to the degenerate nonlinear elliptic partial differential integral equations or variational inequalities of the type which appear as "Bellman" equations. Whether or not the PDE has only a formal meaning, the solutions to the approximating Markov chain problems converge to the desired cost or optimal functional. Owing to many non standard features in such problems and to the nonlinearity of the equations which are to be solved, the decomposition problems for the Markov chain control problems often require special methods. We prove that appropriate adaptations of current decomposition techniques converge under conditions which are typical of many classes of applications. Probabilistic interpretations of the various algorithms in terms of functionals of Markov chains (controlled or not) are used. An important application is to problems in four or more dimensions where, without decomposition, the size and structure of the state space and associated data structures often leads to very poor performance on cache machines.

#### Papers and reports

- [A] The large deviation principle for a general class of queueing systems, I, Trans. of the AMS, Volume 347, pp. 2689-2751.
  - [B] A Weak Convergence Approach to the Theory of Large Deviations, (approx. 430 p.), J. Wiley & Sons, New York, 1996.
  - [C] Large deviation properties of data streams that share a buffer, LCDS Report, Brown University, 1995.
  - [D] Rates of Convergence for Approximation Schemes in Optimal Control, LCDS Report, Brown University, 1994.
- A control problem for a new type of public transportation system, via heavy

traffic analysis. in Stochastic Networks, ed. R. Williams and F. Kelly, Springer, 1995

An effective numerical method for controlling routing in large trunk line networks, 1995, Mathematics in Computation and Simulation, 38, 225–239, 1995

Numerical methods for controlled routing in large trunk line systems via stochastic control theory, ORSA Journal on Computing, 6, 1994, 300–316

Domain decomposition methods for large Markov chain control problems and nonlinear elliptic type problems, to appear SIAM J. on Scientific Computation, 1996

A control problem for a new type of public transportation system, via heavy traffic analysis. in Stochastic Networks, ed. R. Williams and F. Kelly, Springer, 1995

Numerical Methods for Controlled and Uncontrolled Multiplexing and Queueing Systems, IEEE Journal on Special Areas in Communication, 1995

Controlled and optimally controlled multiplexing systems: A numerical exploration, Queueing Theory and Applications, 20, 1996, pages 255–291

Numerical methods in finance. 1995. To appear in a book published by Cambridge University Press.

Programs for controlled multiplexing and ATM-type systems: Documentation, 1995

Documentation of codes for optimal control problems, 1996: Codes are available on the internet.

Large deviations properties of data streams that share a buffer. Submitted to the Annals of Applied Probab.

Software available on the internet

#### **Supported graduate students**

Michelle Boue-McDermid; Female, Hispanic

Kavita Ramanan; Female, Asian

Rachael Wright (Female, Caucasian; ASERT)

John Curran (Male, Caucasian; ASERT)

#### **Other supported personnel**

Dennis Jarvis (Programmer)

#### **Supported PI's**

Paul Dupuis

Harold J. Kushner